AN OBSERVATIONAL STUDY OF KINETIC ENERGY CONVERSIONS IN THE ATMOSPHERE [1, 2]

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ABSTRACT

The total kinetic energy in the atmosphere has been subdivided into four energy reservoirs. The partition of the kinetic energy is accomplished by dividing the total flow into the vertical mean flow (the barotropic component) and the vertical shear flow (the baroclinic component). Each of these components is subdivided into the zonal components and the eddy components.

The complete energy exchange diagram is derived by dividing a given energy conversion into the contribution from the quasi-non-divergent flow and the contribution from the divergent flow. Such a subdivision of the energy conversion is advantageous because the calculations are based on geopotential data.

Calculations have been carried out for five months (January, April, July, October 1962 and January 1963) based on five isobaric surfaces (20, 30, 50, 70, and 85 cb.). The complete energy diagrams are presented for each month together with an averaged diagram representing the annual mean. The results obtained for the four months in 1962 are in good agreement with each other showing not only the same directions of the energy conversions but also a marked annual variation for the major, non-divergent conversions generally with a minimum during the summer season.

The annual mean diagram is compared with the mean diagram obtained in a numerical simulation of the atmospheric general circulation. Good agreement is found in most energy conversions with two major exceptions. The results in the observational study which depend entirely on the mean meridional circulation suffer from the fact that the present data can not give a true picture of the Hadley circulation in the low latitudes. The energy conversion which depends entirely on the eddies is larger in the observational study than in the experimental study. The reason for this discrepancy is ascribed to the lower intensity of the eddies in the experimental study and, in particular, to the lack of energy on the planetary scale in the general circulation experiment.

1. INTRODUCTION

The major components of the energetics of the atmosphere originally formulated by Lorenz [2] have been investigated in great detail during recent years. A comprehensive summary of calculations based on observations has been given by Oort [3] who also has included some results of studies of the general circulation based on long-term numerical integrations of theoretical models of the atmosphere.

The kinetic energy of the atmosphere has been subdivided into the kinetic energy of the zonally averaged flow and the kinetic energy of the remaining flow, the eddies. Several estimates have been made of the energy conversions between the eddy kinetic energy and the zonal kinetic energy. (Starr [8], Saltzman and Fleisher [4], Wiin-Nielsen, Brown, and Drake [12, 13]). A different subdivision of the kinetic energy was introduced by Wiin-Nielsen [10] in close collaboration with Smagorinsky [7] who used this subdivision to describe the energetics of his basic general circulation experiment. The new subdivision consists of dividing the atmospheric flow into the vertically averaged flow (the so-called barotropic component) and the deviation from the vertical mean flow, the vertical shear flow (or the baroclinic component) of the atmospheric flow. The original pilot study by Wiin-Nielsen [10] was later extended (Wiin-Nielsen and Drake [14, 15]) to cover much larger data samples and a greater vertical resolution in addition to an estimate of the contribution from the divergent part of the wind to the energy conversion between the vertical shear flow and the vertical mean flow.

Smagorinsky [7] extended the study of the energy conversions between the different forms of kinetic energy to cover all possible components of the energy transformation between the four forms of energy: (1) the kinetic energy of the zonally averaged vertical mean flow, (2) the kinetic energy of the eddy component of the vertical mean flow, (3) the kinetic energy of the zonally averaged vertical shear flow, and (4) the kinetic energy of the eddy component of the vertical shear flow. We shall in the

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following sections denote these components by the symbols: K_{MZ}, K_{ME}, K_{SZ}, K_{SE}. Numerical values of the many energy conversions between the four energy forms can be found in Smagorinsky's paper [7] for his model.

The same energy conversions have not to the knowledge of the authors been calculated from atmospheric data. It is the main purpose of this paper to present the results of such calculations based on atmospheric height data from five isobaric levels: 85, 70, 50, 30, and 20 cb. The height data are the routine objective analysis carried out by the National Meteorological Center, U.S. Weather Bureau in connection with its short-range numerical prediction program. The data which have been used in several studies of the energetics of the atmosphere (Wiin-Nielsen [11]) were originally made available to us by Dr. George P. Cressman.

There are significant differences between an observational study of atmospheric energy conversions and a study of the same quantities based on a numerical integration of a set of equations which simulates the thermo-hydrodynamic behavior of the atmosphere on a large time scale including modeling of the atmospheric heat sources and the frictional dissipation. One of the main differences is the fact that atmospheric parameters such as the vertical velocity, the horizontal divergence, and the distribution of the atmospheric heat sources escape ordinary synoptic analysis and must be computed by more or less realistic indirect methods while they are readily available as by-products of the numerical integrations of the model equations with an accuracy as great as the electronic computer employed for the experiment will permit. Because of this fact it has been found advantageous to transform the original expressions for the kinetic energy conversions in such a way that we can isolate easily computable quantities such as the horizontal wind and the vertical components of the vorticity in separate integrals. This subdivision is made possible by using well known identities between terms in the hydrodynamic equations. As shown in the earlier paper (Wiin-Nielsen [10]), we can in this way distinguish between terms which will make contributions in a quasi-geostrophic formulation of the atmospheric dynamics and the terms which will contribute only in an atmospheric model based on the primitive equations.

Section 2 of this paper contains an outline of the framework and the formulas which have been used in the calculations presented in this report. The numerical results are given in section 3 for different months and a comparative study is made of the observational results and those obtained by Smagorinsky [7] in his basic numerical experiment.

2. BASIC EQUATIONS FOR THE ENERGY CONVERSIONS

The integrals which are used for the calculation of the energy conversions are naturally very similar to those derived by Smagorinsky [7], although his derivations apply to a two-level representation of the vertical structure of the atmosphere. We begin by defining the wind components used in this study. The vertically averaged wind $\mathbf{v}_M$ is defined by the expression

$$\mathbf{v}_M=\frac{1}{p_0}\int_0^{p_0}\mathbf{v}dp \tag{2.1}$$

in which $\mathbf{v}$ is the horizontal wind vector with components u and v, p is pressure and $p_0=100$ cb.

The deviation of the wind from the vertical mean $\mathbf{v}_M$, the shear vector, is defined by the relation

$$\mathbf{v}_S=\mathbf{v}-\mathbf{v}_M \tag{2.2}$$

Each of the wind components will be subdivided in the zonal average defined by the relation

$$(\ )_Z=\frac{1}{2\pi}\int_0^{2\pi}(\ )d\lambda \tag{2.3}$$

where λ is longitude, while the eddy component is defined as the following deviation:

$$(\ )_E=(\ )-(\ )_Z \tag{2.4}$$

The amount of kinetic energy in the zonal part of the vertical mean flow can be evaluated from the formula:

$$K_{MZ}=\frac{p_0}{g}\int_S\frac{1}{2}(u_{MZ}^2+v_{MZ}^2)dS \tag{2.5}$$

in which $dS=a^2\cos\varphi d\lambda d\varphi$ is the area element (a is the radius of the earth, and φ is latitude), while S is the total area of integration.

The corresponding expressions for the three additional energy forms are

$$K_{ME}=\frac{p_0}{g}\int_S\frac{1}{2}(u_{ME}^2+v_{ME}^2)dS \tag{2.6}$$

$$K_{SZ}=\frac{1}{g}\int_0^{p_0}\int_S\frac{1}{2}(u_{SZ}^2+v_{SZ}^2)dSdp \tag{2.7}$$

and

$$K_{SE}=\frac{1}{g}\int_0^{p_0}\int_S\frac{1}{2}(u_{SE}^2+v_{SE}^2)dSdp \tag{2.8}$$

During the derivations it has been assumed that the boundary conditions for ω are $\omega=0$ for $p=0$ and $p=p_0$. It follows then from the general continuity equation in pressure coordinates that

$$\nabla\cdot\mathbf{v}_M=0 \text{ and } \nabla\cdot\mathbf{v}=\nabla\cdot\mathbf{v}_S \tag{2.9}$$

When we average $\nabla\cdot\mathbf{v}_M=0$ in the zonal direction we obtain:

$$\frac{1}{a\cos\varphi}\frac{\partial v_{MZ}\cos\varphi}{\partial\varphi}=0$$

from which it follows that $v_{MZ}\cos\varphi$ is constant, but since $v_{MZ}\cos\varphi=0$ at the poles we obtain $v_{MZ}=0$ everywhere.

The expression (2.5) reduces therefore to

$$K_{MZ}=\frac{p_0}{g}\int_S \frac{1}{2}u_{MZ}^2 dS \tag{2.10}$$

The complete equations for the energy transformations are obtained by deriving equations for the rate of change of K_{MZ}, K_{ME}, K_{SZ}, and K_{SE}. From (2.10) and (2.6) we obtain

$$dK_{MZ}/dt=\frac{p_0}{g}\int_S u_{MZ}\frac{\partial u_{MZ}}{\partial t}dS \tag{2.11}$$

and

$$dK_{ME}/dt=\frac{p_0}{g}\int_S \left[u_{ME}\frac{\partial u_{ME}}{\partial t}+v_{ME}\frac{\partial v_{ME}}{\partial t}\right]dS \tag{2.12}$$

with expressions analogous to (2.12) obtained from (2.7) and (2.8). It is seen from (2.12) that we must go through the following steps in order to derive an equation for dK_{ME}/dt. First, we must derive the equations governing the local rate of change of u_{ME} and v_{ME}. The next step is to multiply the first of these equations by u_{ME} and the second by v_{ME}. The final step is to add the resulting equations and integrate over the domain S. A similar procedure must be followed in order to obtain equations for dK_{SZ}/dt and dK_{SE}/dt. The derivation of these equations is rather straightforward although laborious and cumbersome because of the many different subscripts and the two different averaging procedures which are being used. The detailed derivation of the equations is probably of no great interest to the majority of the readers. We shall therefore in this paper be satisfied by giving the final results. The reader interested in the details is referred to a technical report with the same title as this paper available from the Department of Meteorology and Oceanography, the University of Michigan.

The main results of the detailed derivations as outlined above can be expressed in terms of energy conversion C and dissipation D in symbolic form in the following four equations:

$$\frac{dK_{MZ}}{dt}=C(K_{ME}, K_{MZ})+C(K_{SE}, K_{MZ}) +C(K_{SZ}, K_{MZ})-D(K_{MZ}) \tag{2.13}$$

$$\frac{dK_{ME}}{dt}=-C(K_{ME}, K_{MZ})+C(K_{SZ}, K_{ME})+C(K_{SE}, K_{ME}) +C(K_{SE}, [K_{SZ}], K_{ME})-D(K_{ME}) \tag{2.14}$$

$$\frac{dK_{SZ}}{dt}=C(A_Z, K_{SZ})+C(K_{SE}, K_{SZ})-C(K_{SZ}, K_{ME}) -C(K_{SZ}, K_{MZ})-D(K_{SZ}) \tag{2.15}$$

and

$$\frac{dK_{SE}}{dt}=C(A_E, K_{SE})-C(K_{SE}, K_{SZ})-C(K_{SE}, K_{ME}) -C(K_{SE}, K_{MZ})-C(K_{SE}, [K_{SZ}], K_{ME})-D(K_{SE}) \tag{2.16}$$

It should be mentioned that the derivation of the equations (2.13)–(2.16) is based on the complete spherical, hydrostatic equations using pressure as the vertical coordinate.

We notice first of all that each of the energy conversions involving only kinetic energy components appears twice in the equations with opposite signs. This property insures that the change in total kinetic energy will satisfy the usual relations:

$$\frac{dK}{dt}=C(A, K_S)-D(K) \tag{2.17}$$

where $C(A, K_S)=C(A_Z, K_{SZ})+C(A_E, K_{SE})$, and where $D(K)=D(K_{MZ})+D(K_{ME})+D(K_{SZ})+D(K_{SE})$.

Figure 1 shows in another symbolic form the content of equations (2.13)–(2.16). The hexagonal boxes in figure 1 give the symbolic name of the energy conversion together with the integrand in the energy conversion integral. Each integrand has to be integrated with respect to pressure, latitude, and longitude. If an arbitrary integrand is denoted by I we may write

$$C=\frac{1}{g}\int_0^{p_0}\int_{\varphi_1}^{\varphi_2}\int_0^{2\pi} Ia^2\cos\varphi d\lambda d\varphi dp. \tag{2.18}$$

The following formulas contain the integrals for the seven energy conversions calculated in this study:

$$C(K_{ME}, K_{MZ})=\frac{2\pi a p_0}{g}\int_{\varphi_1}^{\varphi_2}(u_{ME}v_{ME})_Z\cos^2\varphi\frac{\partial}{\partial\varphi}\left(\frac{u_{MZ}}{\cos\varphi}\right)d\varphi \tag{2.19}$$

$$C(K_{SE}, K_{MZ})=\frac{2\pi a}{g}\int_0^{p_0}\int_{\varphi_1}^{\varphi_2}(u_{SE}v_{SE})_Z\cos^2\varphi\frac{\partial}{\partial\varphi}\left(\frac{u_{MZ}}{\cos\varphi}\right)d\varphi dp \tag{2.20}$$

$$C(K_{SZ}, K_{MZ})=\frac{2\pi a}{g}\int_0^{p_0}\int_{\varphi_1}^{\varphi_2}(u_{SZ}v_{SZ})\cos^2\varphi\frac{\partial}{\partial\varphi}\left(\frac{u_{MZ}}{\cos\varphi}\right)d\varphi dp \tag{2.21}$$

$$C(K_{SZ}, K_{ME})=\frac{2\pi a^2}{g}\int_0^{p_0}\int_{\varphi_1}^{\varphi_2}[\{(u_{ME}\zeta_{SE})_Z v_{SZ}-(v_{ME}\zeta_{SE})_Z u_{SZ}\} -\{(u_{ME}u_{SE})_Z+(v_{ME}v_{SE})_Z\}\nabla\cdot\mathbf{v}_{SZ}]\cos\varphi d\varphi dp \tag{2.22}$$

$$C(K_{SE}, K_{ME})=\frac{a^2}{g}\int_0^{p_0}\int_{\varphi_1}^{\varphi_2}\int_0^{2\pi}[(u_{ME}v_{SE}-v_{ME}u_{SE})\zeta_{SE} -(u_{ME}u_{SE}+v_{ME}v_{SE})\nabla\cdot\mathbf{v}_{SE}]\cos\varphi d\lambda d\varphi dp \tag{2.23}$$

$$C(K_{SE}, K_{SZ})=\frac{2\pi a^2}{g}\int_0^{p_0}\int_{\varphi_1}^{\varphi_2}\Big[(v_{SE}\zeta_{ME})_Z u_{SZ}-(u_{SE}\zeta_{ME})_Z v_{SZ} -\left(\omega_{ME}\frac{\partial u_{SE}}{\partial p}\right)_Z u_{SZ}-\left(\omega_{ME}\frac{\partial v_{SE}}{\partial p}\right)_Z v_{SZ}\Big]\cos\varphi d\varphi dp \tag{2.24}$$

and

$$C(K_{SE}, [K_{SZ}], K_{ME})=\frac{2\pi a^2}{g}\int_0^{p_0}\int_{\varphi_1}^{\varphi_2}[(u_{ME}v_{SE}-v_{ME}u_{SE})_Z\zeta_{SZ} (u_{ME}\nabla\cdot\mathbf{v}_{SE})_Z u_{SZ}-(v_{ME}\nabla\cdot\mathbf{v}_{SE})_Z v_{SZ}]\cos\varphi d\varphi dp \tag{2.25}$$

It will be noticed that the integrals (2.19), (2.20), and (2.21) are obtained after an integration by parts using the lateral boundary condition. This procedure has been followed in order to obtain a form which is analogous to

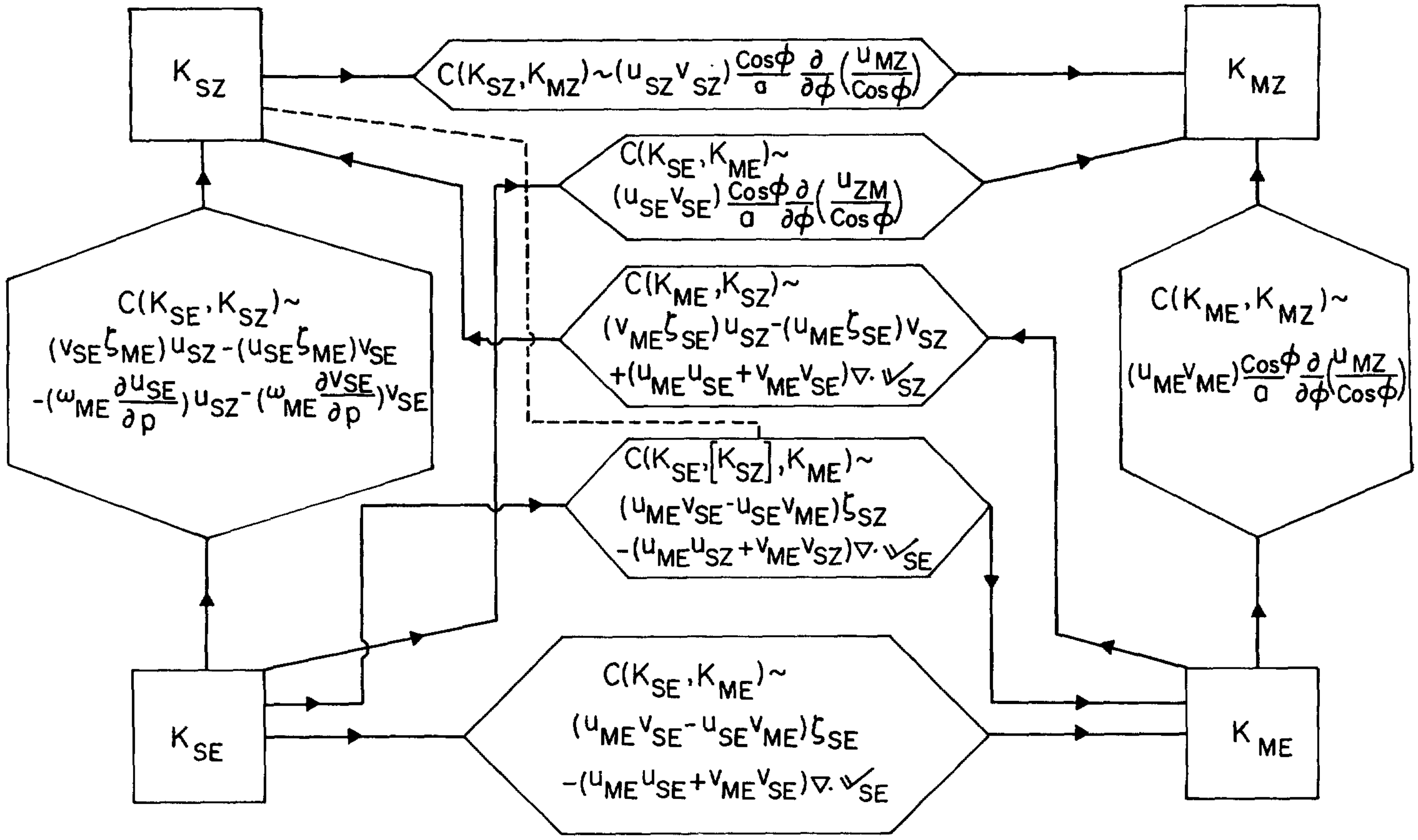

FIGURE 1.—Schematic energy diagram showing all energy conversions between the four energy forms: K_{SZ}, K_{SE}, K_{MZ}, and K_{ME}. The hexagonal boxes show the energy conversion together with the integrand in the energy conversion integral.

the form used earlier to compute the energy conversion from the eddies to the zonal flow (Wiin-Nielsen, Brown, and Drake [12], [13]). The physical interpretation of energy conversions of this kind has been discussed by several authors (Kuo [1], Starr [8], Wiin-Nielsen et al. [12, 13]). The kinetic energy of the zonally averaged vertical mean flow, see equation (2.13), will increase if we have a positive correlation between the proper momentum transport and the meridional shear of the zonal. vertical mean flow u_{MZ}. It is obvious that the three integrals (2.19)–(2.21) represent physical processes of the same nature, but while a numerical estimate of the first two integrals ((2.19) and (2.20)) can be obtained from the non-divergent assumption it is evident that the third integral (2.21) depends entirely on a divergent wind component (v_{SZ}). If we want to evaluate the integral (2.21) we must necessarily be able to calculate the vertical velocity, the horizontal divergence, and thereby the divergent wind components, in particular v_{SZ}. The procedure which has been used for this purpose will be described later in this section of the paper.

The integrals (2.22)–(2.25) consist of terms depending essentially on the rotational part of the flow and other terms which only can be evaluated when we know the irrotational components of the flow. According to our present experience the rotational components will give the larger contributions to the energy conversions simply because the vertical component of vorticity in general is large compared to the horizontal divergence. We have selected the forms of the energy conversions appearing in (2.22)–(2.25) in order to distinguish between contributions of the first and the second kind. If a part of an energy conversion requires any component of the mean meridional circulation, a divergence or a vertical velocity, it will thus be classified as a divergent component of the energy conversion in question and denoted by a subscript D. The remaining part of the energy conversion will be classified as a non-divergent component and will have a subscript ND.

We find by inspection of (2.19)–(2.25) that $C(K_{ME}, K_{MZ}) = C_{ND}(K_{ME}, K_{MZ})$, $C(K_{SE}, K_{MZ}) = C_{ND}(K_{SE}, K_{MZ})$, while $C(K_{SZ}, K_{MZ}) = C_D(K_{SZ}, K_{MZ})$. The remaining energy conversions ((2.22)–(2.25)) will have both non-divergent and divergent parts. The non-divergent parts are given in the following expressions:

$$C_{ND}(K_{SZ}, K_{ME}) = -\frac{2\pi a^2}{g}\int_0^{p_0}\int_{\varphi_1}^{\varphi_2}(v_{ME}\zeta_{SE})_Z u_{SZ}\cos\varphi\, d\varphi\, dp \tag{2.26}$$

$$C_{ND}(K_{SE}, K_{ME}) = \frac{a^2}{g}\int_0^{p_0}\int_{\varphi_1}^{\varphi_2}\int_0^{2\pi}(u_{ME}v_{SE} - v_{ME}u_{SE})\,\zeta_{SE}\cos\varphi\, d\lambda\, d\varphi\, dp \tag{2.27}$$

$$C_{ND}(K_{SE}, K_{SZ}) = \frac{2\pi a^2}{g}\int_0^{p_0}\int_{\varphi_1}^{\varphi_2}(v_{SE}, \zeta_{ME})_Z u_{SZ}\cos\varphi\, d\varphi\, dp \tag{2.28}$$

and

$$C_{ND}(K_{SE}, [K_{SZ}], K_{ME})=\frac{2\pi a^2}{g}\int_0^{p_0}\int_{\varphi_1}^{\varphi_2} (u_{ME}v_{SE}-v_{ME}u_{SE})_Z\cdot\zeta_{SZ}\cos\varphi d\varphi dp \quad (2.29)$$

The divergent parts of the same energy conversions are naturally the remaining terms in (2.22)–(2.25). It should be mentioned that the nondivergent components, C_{ND}, will be present in a quasi-geostrophic model of the atmospheric motion, while we must use the primitive equations to incorporate the components C_D as an integral part of the model.

One of the energy conversions has a special interpretation. The conversion (2.25) (and its nondivergent part (2.29)) represent an energy conversion from K_{SE} to K_{ME} in which K_{SZ} participates although it remains unchanged. Such an energy conversion is called catalytic by Smagorinsky [7] in close analogy with chemical terminology. The concept of a catalytic conversion is very essential in the formulation of the energy diagram, figure 1, which is based on the equations stated in this section. It explains why the energy conversion $C(K_{SE}, K_{SZ})$ contains quantities with subscript ME, and why $C(K_{SZ}, K_{ME})$ includes parameters with subscripts SE. The reason for this special condition is exactly that the zonal shear flow energy, K_{SZ}, acts as a catalyst for the energy conversion $C(K_{SE}, K_{ME})$. Following Smagorinsky [7] we have written the noncatalytic parts of the total energy conversions $C(K_{SE}, K_{SZ})$ and $C(K_{SZ}, K_{ME})$. Hence the appearance of the additional subscripts. Smagorinsky [7] gives the necessary formulas to find the catalytic and noncatalytic parts of a given energy conversion.

The purpose of this investigation is to calculate the seven energy conversions (2.19)–(2.25) from observed data. The energy conversions $C(A_Z, K_{SZ})$ and $C(A_E, K_{SE})$ appearing in (2.15) and (2.16) will not be calculated because they have been investigated in detail in observational studies before (Saltzman and Fleisher [5], [6], Wiin-Nielsen [9]). The frictional dissipations appearing in (2.13)–(2.16) will furthermore not be included in this paper.

The nondivergent energy components create no special problem in a calculation based on atmospheric data. Only height data were available for the calculations reported in this study. From the height data we have obtained a so-called geostrophic streamfunction ψ by solving the equation

$$\nabla^2\psi=\frac{1}{f}\nabla^2\varphi-\frac{1}{f^2}\nabla f\cdot\nabla\varphi \quad (2.30)$$

The vorticity has been computed as $\zeta=\nabla^2\psi$ while the wind components have been obtained from $u=-a^{-1}\partial\psi/\partial\varphi$ and $v=(a\cos\varphi)^{-1}\partial\psi/\partial\lambda$. The streamfunction ψ was first obtained by solving (2.30) at the five data levels mentioned in the introduction. The four components, ψ_{MZ}, ψ_{ME}, ψ_{SZ}, and ψ_{SE} were next computed using the definitions (2.1) to (2.4). The remaining calculations necessary to obtain all the components C_{ND} were completed using a computational procedure as described by Wiin-Nielsen and Drake [14].

The energy conversions C_D require a knowledge of a vertical velocity, a divergence, or the component v_{SZ} of the mean meridional circulation. In order to obtain a numerical estimate of the conversions C_D we have followed the procedure, outlined by Wiin-Nielsen and Drake [15], of computing the vertical velocity ω from the so-called ω-equation, obtaining the horizontal divergence from the continuity equation, and then calculating ω_{ME}, $\nabla\cdot\mathbf{v}_{SE}$, and $\nabla\cdot\mathbf{v}_{SZ}$ using the averaging procedures. Finally, the component v_{SZ} was obtained by integration of the continuity equation for the zonally averaged flow

$$\frac{1}{a\cos\varphi}\frac{\partial v_{SZ}\cos\varphi}{\partial\varphi}=\nabla\cdot\mathbf{v}_{SZ} \quad (2.31)$$

Equation (2.31) can be integrated starting from the North Pole where $v_{SZ}\cos\varphi=0$.

3. RESULTS OF ENERGY CONVERSION CALCULATIONS

The energy conversion integrals which have been derived in the previous section of this paper have been calculated from observations using height data at five levels (85, 70, 50, 30, and 20 cb.) for five different months (January, April, July, October, 1962 and January 1963). The results of the calculations will be described in this section. It is pertinent to mention that all the calculations have been carried out on a daily basis while only monthly averages will be presented in this paper.

The averaged monthly values for all energy conversions are presented in figures 2–6. It is known from earlier studies (Wiin-Nielsen, Brown, and Drake, [13]) that January 1963 is an abnormal month with respect to the conversion of kinetic energy from the atmospheric eddies to the zonal flow. We must therefore expect that abnormal behavior to show up in the present calculations, and we shall therefore discuss the results from this month separately later in this section. The results from the different months during 1962 show a remarkable degree of similarity. We notice first of all that in the cases of contribution from both the non-divergent and the divergent components of the flow, the contribution C_D is in general much smaller than the contribution C_{ND}. The only exception is the catalytic conversion of $C(K_{ME}, [K_{SZ}], K_{SE})$ where the contributions C_D and C_{ND} are of equal order of magnitude, but they are both small and usually of the opposite sign.

It should furthermore be noticed that the directions of the energy conversions agree during all four months, while January 1963 gives some exceptions. The major energy conversion which contributes to the maintenance of the kinetic energy of K_{SE} must be the energy conversion $C(A_E,K_{SE})$. During the months in 1962 there was furthermore a *small* positive contribution from the catalytic conversion $C(K_{ME}, [K_{SZ}], K_{SE})$. The major output of energy from the reservoir K_{SE} goes to the kinetic energy K_{SZ} through the conversion $C(K_{SE}, K_{SZ})$, while a

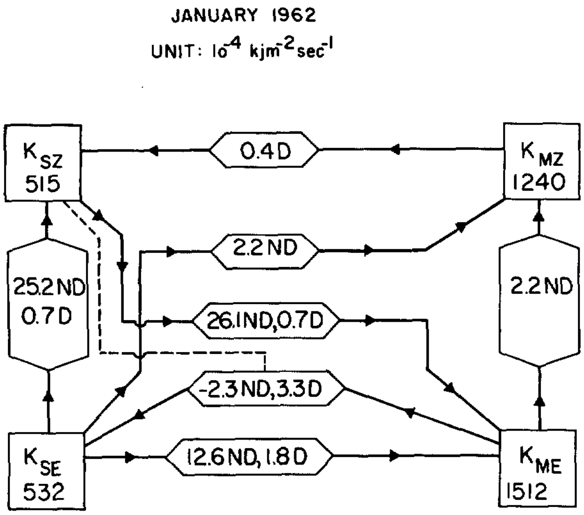

FIGURE 2.—Energy conversion diagram for January 1962. The units are kj. m.$^{-2}$ for the amounts of energy and 10^{-4} kj. m.$^{-2}$ sec.$^{-1}$ for energy conversion. *ND* is an energy conversion which would be present in a quasi-non-divergent formulation, while *D* depends on the divergent component of the wind.

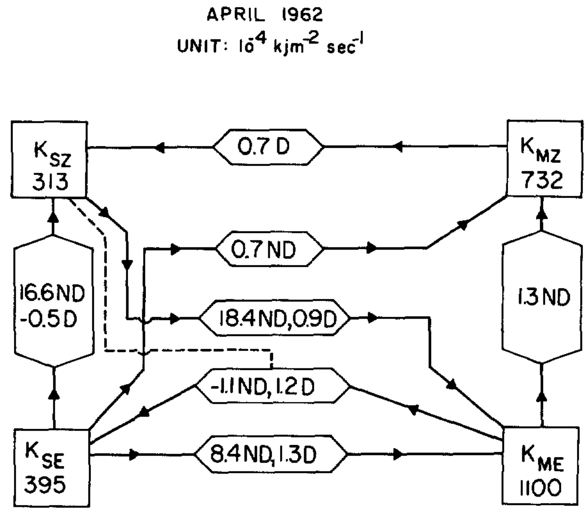

FIGURE 3.—Energy conversion diagram for April 1962.

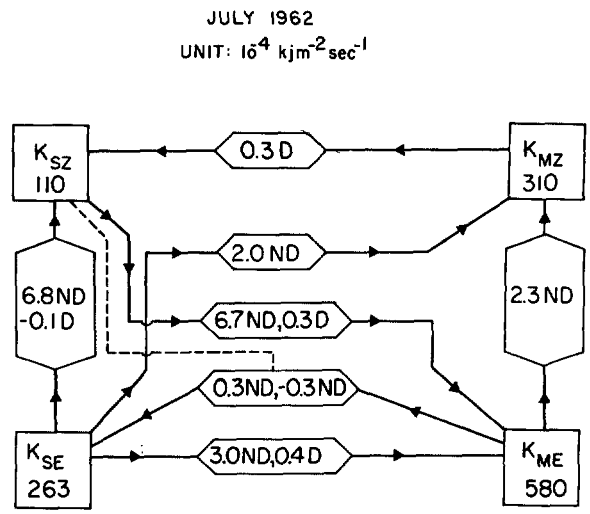

FIGURE 4.—Energy conversion diagram for July 1962.

relatively small amount goes to K_{MZ} through the conversion $C(K_{SE}, K_{MZ})$.

Turning our attention to K_{SZ} we find that the conversion $C(K_{SZ}, K_{ME})$ at all times is of the same order of magnitude as the conversion $C(K_{SE}, K_{SZ})$. These conversions are the only major energy conversions in connection with K_{SZ}. The minor conversions are $C(K_{MZ}, K_{SZ})$ and, presumably, $C(K_{SZ}, A_Z)$, although the last energy conversion has not been computed in this study. However, it has been estimated in earlier studies (Wiin-Nielsen, [9], Saltzman and Fleisher, [5, 6]) and has been found to be small.

We have already mentioned that K_{ME} receives a large amount of energy through the conversion $C(K_{SZ}, K_{ME})$. In addition, it can be seen from the figures that an appreciable amount of energy is added to the eddies of the vertical mean flow (the barotropic waves) through a direct conversion of energy from K_{SE}, the eddies in the vertical shear flow. K_{ME} is the energy component which contains the largest amount of energy, but at the same time the component which receives the largest amount of energy through energy conversion from other reservoirs of kinetic energy. K_{ME} loses a small amount of energy through the catalytic conversion $C(K_{ME}, [K_{SZ}], K_{SE})$ and a somewhat larger amount to the reservoir K_{MZ} through the conversion $C(K_{ME}, K_{MZ})$.

Finally the energy reservoir K_{MZ} contains the next to the largest amount of energy. However, it receives only relatively small amounts of energy through the energy conversions $C(K_{SE}, K_{MZ})$ and $C(K_{ME}, K_{MZ})$, while a small amount of energy is lost from K_{MZ} through the conversion $C(K_{MZ}, K_{SZ})$.

The energy conversion diagram for January 1963 is reproduced in figure 6. By a comparison between figure 6 and the figures 2, 3, 4, and 5, we find two major differences. The energy conversion between the reservoirs K_{MZ} and K_{ME} has a direction which is opposite to the direction we found for all the months investigated during 1962. Furthermore, the direction has changed for the catalytic conversion during this month. We observe finally that the conversion $C_D(K_{MZ}, K_{SZ})$ has a value somewhat larger than calculated during the year 1962. The present calculations are in agreement with earlier calculation (Wiin-Nielsen, Brown, and Drake [13]) with respect to the energy conversions between the eddies and the zonal flow.

Although the results from the individual months are most interesting to observe, there are such large changes between the months in the amounts of energy and in the different energy conversions that only the annual average can approach a steady state. We have therefore averaged the results for the four individual months in 1962 to produce an approximation to the annual average. The resulting diagram is reproduced as figure 7 of this paper. Because of the great similarity between the diagrams for the individual months with respect to the directions of the

OCTOBER 1962
UNIT: 10^{-4} kjm^{-2} sec^{-1}

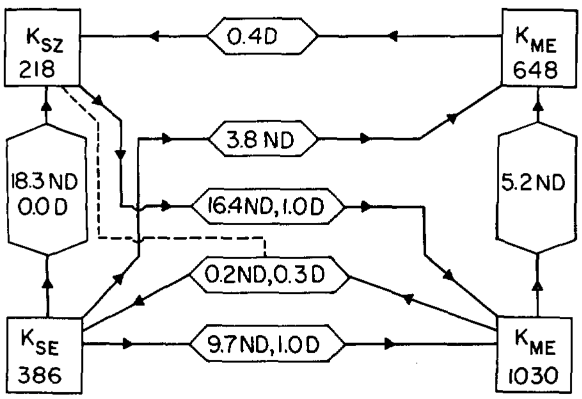

FIGURE 5.—Energy conversion diagram for October 1962.

ANNUAL MEAN 1962
UNIT: 10^{-4} kjm^{-2} sec^{-1}

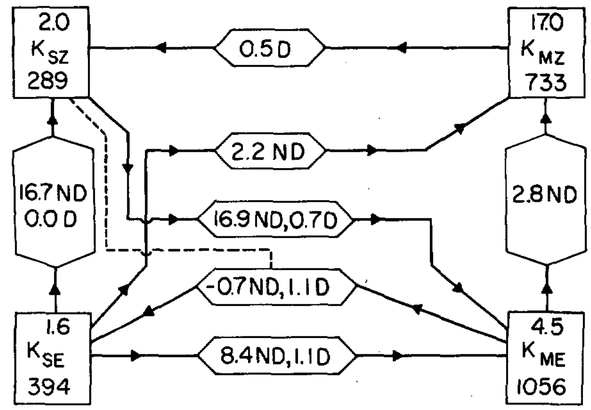

FIGURE 7.—Energy conversion diagram obtained by averaging the results for January, April, July, and October 1962.

JANUARY 1963
UNIT: 10^{-4} kjm^{-2} sec^{-1}

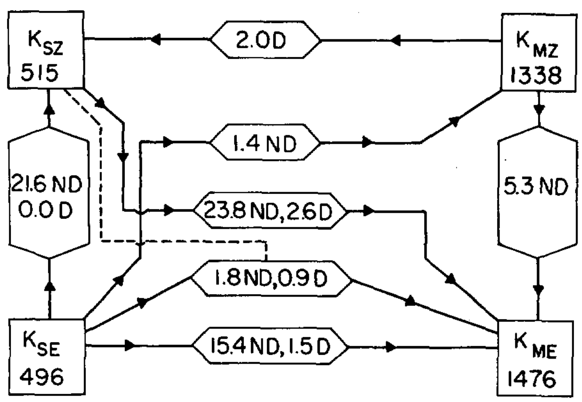

FIGURE 6.—Energy conversion diagram for January 1963.

SMAGORINSKY'S DIAGRAM
UNIT: 10^{-4} kjm^{-2} sec^{-1}

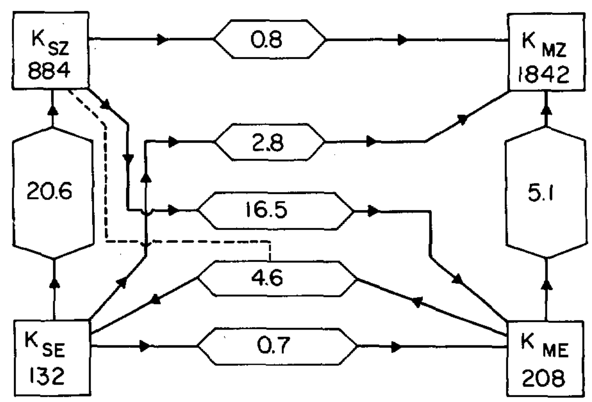

FIGURE 8.—Energy conversion diagram from Smagorinsky [7].

energy conversions, we find no changes in the directions in the annual averages, but the large seasonal variations in the conversions have been removed.

One of the main reasons for the construction of figure 7 is to enable us to make a comparison with the results obtained by Smagorinsky [7] from his basic numerical experiment concerning the general circulation of the atmosphere. Figure 8 has been constructed using his results which have been converted to our units and simplified slightly by combining the energies of the zonal and meridional components of the zonal vertical shear flow.

A comparison of figure 6 and figure 8 shows many similarities between the two calculations. We observe first of all that the largest energy conversions in figure 7, i.e., $C(K_{SE}, K_{SZ})$ and $C(K_{SZ}, K_{ME})$, are also the largest conversions in figure 8. There is agreement between the directions of the energy conversions in the two diagrams except for the small conversion $C_D(K_{SZ}, K_{MZ})$ which depends entirely on the mean meridional circulation. One of the reasons that we find a conversion from K_{MZ} to K_{SZ} opposite to Smagorinsky [7] is undoubtedly the fact that our calculation does not include data from the very low latitudes. It is by now well established that a Hadley circulation exists in the low latitudes with negative values of v_Z at low altitudes and positive values of v_Z at high altitudes. Since u_Z and also u_{SZ} are negative at low elevations but positive at high elevations, we will find a positive momentum transport by the mean meridional circulation in the Hadley cell, i.e., $u_{SZ}v_{SZ}>0$. Furthermore, it is easily seen from mean meridional cross-sections that $\partial(\cos^{-1}\varphi u_{MZ})/\partial\varphi>0$ in the low latitudes, and it is therefore evident that the contribution from the Hadley cell to the conversion $C(K_{SZ}, K_{MZ})$ will be positive and perhaps large enough to change the directions of the conversion as found in our calculation. Nevertheless, we will find that the conversion $C(K_{SZ}, K_{MZ})$ is numerically small.

Although we find agreement in the directions of the remaining energy conversion in figure 7 and figure 8, there is one major discrepancy in the order of magnitude of the energy conversions in the two calculations. We find $C(K_{SE}, K_{ME})=9.5\times10^{-4}$kj.m.$^{-2}$ sec.$^{-1}$, while Smagorinsky [7] finds 0.7×10^{-4}kj.m.$^{-2}$ sec.$^{-1}$ for the same conversion. The much smaller number found in figure 8 is undoubtedly due to the smaller intensity of the eddy motion in the numerical experiment which forms the basis for the numbers appearing in figure 8, combined with the fact that no waves of a truly planetary scale were formed in the numerical experiment. One would therefore expect that the difference will disappear when the mountain effect and the ocean-continent effects are introduced in a numerical experiment of the general circulation. Such modifications of general circulation experiments are apparently being performed at the present time.

There is in general good agreement between the numbers appearing in figure 7 and figure 8 in the remaining energy conversions. The differences which are observed can easily be due to the specific sample of four months used in our observational study and to the fact that the numerical experiment with which we have compared has a lateral wall in the higher latitudes.

We shall finish this section with a computation of the residence times in the four kinetic energy reservoirs in figure 7. Such a calculation is hampered by the fact that our energy diagram is incomplete, because we have no estimates of the frictional dissipations. The estimate of the residence time T in the reservoir K_{MZ} can be obtained because we know the total input of energy, i.e., $C(K_{SE}, K_{MZ})$ and $C(K_{ME}, K_{MZ})$, which amounts to 5×10^{-4} kj. m.$^{-2}$ sec.$^{-1}$ With a total amount of $K_{MZ}=733$ kj. m.$^{-2}$, we find $T(K_{MZ})=17$ days, while the dissipation $D(K_{MZ})=4.5\times10^{-4}$ kj. m.$^{-2}$ sec.$^{-1}$ if figure 7 represents a steady state. We find in a similar way that $T(K_{ME})=4.5$ days because the net input is 27.1×10^{-4} kj. m.$^{-2}$ sec.$^{-1}$ and $K_{ME}=1056$ kj. m.$^{-2}$ The dissipation turns out to be 23.9×10^{-4} kj. m.$^{-2}$ sec.$^{-1}$ if we have a steady state.

The reservoir $K_{SZ}=289$ kj. m.$^{-2}$ has a total input of 17.2×10^{-4} kj. m.$^{-2}$ sec.$^{-1}$ assuming that the conversion $C(K_{SZ}, A_Z)<0$ as has been found in most observational studies (Wiin-Nielsen, [11]). We find therefore a residence time of $T(K_{SZ})=2$ days. The residence time in K_{SE} is the most difficult to evaluate because we have no information in our study of the energy input $C(A_E, K_{SE})$. On the other hand, we also lack information concerning the frictional dissipation. We are therefore forced to assume that $D(K_{SE})$ is small, and that the energy outflow can be measured as $C(K_{SE}, K_{SZ})+C(K_{SE}, K_{MZ})+C(K_{SE}, K_{ME})=28.4\times10^{-4}$ kj. m.$^{-2}$ sec,$^{-1}$ which gives $T(K_{SE})=1.6$ days. This value is probably too high because of the neglect of $D(K_{SE})$.

The computed residence times have been entered in the four reservoirs in figure 7. Although each of them has a degree of uncertainty we can conclude that the smallest residence times are found in the baroclinic components of the flow, K_{SZ} and K_{SE}, while the barotropic components K_{MZ} and K_{ME} have residence times which are considerably longer.

The calculations in this paper are naturally to some extent overlapping some of our earlier calculations of the energy exchange between the baroclinic and barotropic components of the kinetic energy. While the emphasis in our earlier studies (Wiin-Nielsen and Drake [14], [15]) was on the spectral distributions of the different forms of energy and the energy conversions, we have in this study emphasized the more detailed breakdown of the many possible energy conversions mainly to provide an observational study which can be compared with numerical studies of the general circulation. However, by recombining the energy conversions calculated in the investigation we can form some of the conversions calculated earlier. Ideally, one should naturally obtain the same numbers since we are using the same data and the same quasi-geostrophic framework. It should, however, be kept in mind that different computational procedures have been used. In the earlier calculations we used the contributions from the first 15 Fourier components to calculate the total conversion. This procedure requires a calculation of many different Fourier coefficients at a large number of latitudes, while the present calculations are based on a more direct finite-difference approach. Since the two calculations will have different truncation errors it is unlikely that complete agreement can be obtained. The general order of magnitude and the variation from month to month should, however, agree if both procedures are valid.

It was found in our earlier study (Wiin-Nielsen and Drake [15]) that the conversions $C_D(K_S, K_M)$ were insignificantly different from zero. The present study shows the same fact. It is therefore of no importance to obtain agreement on any of the divergent conversions as long as they are small.

We shall first investigate the non-divergent conversion $C_{ND}(K_S, K_M)$. The results of the present and previous calculations are given in table 1 of this paper for comparative purposes. The values from the previous study are taken from table 1 of Wiin-Nielsen and Drake [14], while the values from the present study are obtained by adding the non-divergent parts of the energy conversions $C(K_{SE}, K_{MZ})$, $C(K_{SZ}, K_{ME})$, $C(K_{SE}, K_{ME})$ and $C(K_{SE}, [K_{SZ}], K_{ME})$. It is seen from table 1 that a very good agreement is obtained in spite of the radically different numerical procedures which were used. The comparison brings up an interesting and important point. In our previous study [14] we proposed to reduce the calculated values of $C_{ND}(K_S, K_M)$ by about 20 percent, and the corrected numbers are given in table 2 of our previous paper The recommended reduction was based on test calculations for only three days. It is seen from table 1 of the present paper that a 20 percent reduction is unjustified. We are therefore forced to conclude that the annual mean value of $C_{ND}(K_S, K_M)$ is about 28×10^{-4} kj. m.$^{-2}$ sec.$^{-1}$ as compared to 16×10^{-4} kj. m.$^{-2}$ sec.$^{-1}$ obtained by Smagorinsky [7]. The rather large value of $C(K_S, K_M)=28\times10^{-4}$ kj. m.$^{-2}$ sec.$^{-1}$ must now be compared with the

TABLE 1.—*Present and previous values of* $C_{ND}(K_S, K_M)$. *Unit:* 10^{-4}kj.m.$^{-2}$sec.$^{-1}$

	Present study	Previous study
January 1962	43.2	46.5
April 1962	28.6	28.8
July 1962	11.4	12.4
October 1962	29.7	29.6
January 1963	42.4	41.6
Mean value 1962	28.2	29.3

TABLE 2.—*Present and previous values of* $C_{ND}(K_E, K_Z)$. *Unit:* 10^{-4}kj.m.$^{-2}$sec.$^{-1}$

	Present study	Previous study
January 1962	3.5	1.0
April 1962	0.2	0.3
July 1962	4.4	3.5
October 1962	10.9	8.4
January 1963	-6.1	-9.1
Mean value 1962	4.8	3.3

best possible estimate of $C(A, K_S)$. A summary of existing estimates have been made by Oort [3] and Wiin-Nielsen [11]. The value for the winter season is 27.3 $\times 10^{-4}$ kj.m.$^{-2}$ sec.$^{-1}$, while the summer value is 9.5×10^{-4} kj. m.$^{-2}$ sec.$^{-1}$ for $C(A, K_S)$ (see figs. 8 and 9 in Wiin-Nielsen [11]). The annual mean value is therefore 18.4 $\times 10^{-4}$ kj. m.$^{-2}$ sec.$^{-1}$ which is considerably smaller than the value for $C(K_S, K_M)$. Since the numerical value for $C(K_S, K_M)$ does not depend critically on the vertical velocities or the horizontal divergence, and we therefore have greater confidence in $C(K_S, K_M)$ than in $C(A, K_S)$, we must conclude that our present estimates of $C(A, K_S)$ are too small, which agrees with the general impression that the vertical velocities produced by the short-range numerical prediction models are underestimated.

We may in a similar way combine some of the energy conversions calculated in this study to find $C(K_E, K_Z)$, the total kinetic energy conversion from the eddies to the zonal flow. The non-divergent part of the energy conversion $C_{ND}(K_E, K_Z)$ can be found from the diagrams by adding $C_{ND}(K_{SE}, K_{SZ})$, $C_{ND}(K_{SE}, K_{MZ})$, $C_{ND}(K_{ME}, K_{SZ})$, and $C_{ND}(K_{ME}, K_{MZ})$. The resulting numbers should be compared with $C_{ND}(K_E, K_Z)$ computed in the wave number regime from the same data by Wiin-Nielsen, Brown, and Drake [12], [13]. The values for each month are reproduced in table 2 of this paper.

The numbers appearing in table 2 are not as close to each other as the results listed in table 1, although the general trend compares favorably. The reason for the discrepancies is most likely that the contributions from the different pressure levels are often of opposite signs as can be seen from table 2 of Wiin-Nielsen, Brown, and Drake [13]. Small deviations in the results from the individual pressure levels may in this situation give larger deviations in the total value. It is naturally also possible that the conversions which are parts of $C_{ND}(K_E, K_Z)$ are more sensitive to the different truncation and finite-difference errors.

TABLE 3.—*Monthly means (m) and standard deviations (sd) for all kinetic energy conversions. Unit:* 10^{-4}kj.m.$^{-2}$sec.$^{-1}$

	Jan. 1962		April 1962		July 1962		Oct. 1962		Jan. 1963	
	m	*sd*	*m*	*sd*	*m*	*sd*	*m*	*sd*	*m*	*sd*
$C_D(K_{SZ}, K_{MZ})$	−0.4	0.7	−0.7	0.5	−0.2	0.2	−0.4	0.4	−2.0	1.0
$C_{ND}(K_{SE}, K_{MZ})$	+2.2	2.3	+0.7	2.0	+2.0	1.4	+3.8	1.6	+1.4	2.5
$C_{ND}(K_{ME}, K_{MZ})$	+2.2	6.1	+1.2	3.7	+2.3	3.4	+5.2	4.1	−5.3	7.4
$C_{ND}(K_{SE}, K_{SZ})$	+25.2	10.4	+16.6	9.4	+6.8	2.9	+18.3	8.6	+21.6	10.1
$C_D(K_{SE}, K_{SZ})$	+0.7	1.6	−0.5	0.9	−0.0	0.3	+0.0	0.8	+0.0	1.7
$C_{ND}(K_{ME}, K_{SZ})$	−26.1	11.5	−18.4	9.2	−6.7	2.5	−16.4	8.2	−23.8	9.4
$C_D(K_{ME}, K_{SZ})$	−0.7	1.5	−0.9	0.7	−0.3	0.3	−1.0	0.8	−2.6	2.0
$C_{ND}(K_{SE}, K_{ME})$	+12.6	5.6	+8.4	3.1	+3.0	2.9	+9.7	6.8	+15.4	6.9
$C_D(K_{SE}, K_{ME})$	+1.8	3.2	+1.3	2.0	+0.4	0.9	+1.0	2.3	+1.5	3.4
$C_{ND}(K_{SE}, [K_{SZ}], K_{ME})$	+2.2	2.3	+1.1	1.7	+0.3	1.0	−0.2	1.9	+1.8	2.5
$C_D(K_{SE}, [K_{SZ}], K_{ME})$	−3.3	2.6	−1.2	2.0	−0.3	0.3	−0.3	1.3	+0.9	2.5

Although we have been satisfied to discuss the mean values for the individual months together with the annual mean based on four months of data in this paper, it is pertinent to say a few words about the time series which form the basis for the computed mean values. A visual inspection of the energy conversions as a function of time during any month (not reproduced) shows a considerable scatter around the monthly mean values. To give the reader some idea of the variations during the different months we are giving table 3 which contains the monthly mean values and the standard deviations with respect to the monthly averages. It is seen that only the numerically large, non-divergent conversions have standard deviations smaller than the mean values, while the standard deviation is larger than the mean in all divergent cases. We need thus rather long time series to obtain significant mean values. An inspection of the time series for the different months indicates considerable variability in most energy conversions as is also seen from the standard deviations in table 3. A detailed analysis of the daily variation can be made only when calculations are extended to cover many consecutive months. It is, however, evident that the kinetic energy conversions are more variable with respect to the direction of the energy conversion than the energy conversions involving the available potential energy.

All calculations reported in this paper are based on routinely analyzed geopotential data. The operational, objective analysis procedure involves quite a few modifications of the data. In addition to smoothing and interpolation it is found necessary to use the observed winds as if they were almost geostrophic, and to employ a numerical forecast from the preceding observation time as a first guess to the analysis of the geopotential. It is conceivable that these operational procedures although perhaps acceptable for purposes of short-range prediction will create systematic errors which in turn will influence the results of calculations as described in this paper. My research associate, Dr. E. Holopainen has made a comparative study (unpublished) of the transports of momentum and sensible heat as calculated from objective analyses, from subjective analyses, and from observed winds by different investigators. Although no two studies cover the same time periods, and a direct comparison therefore is impossible, it is nevertheless interesting to notice that the comparisons show that the momentum and sensible heat transports computed from the objective analyses are systematically larger in the middle latitudes than the transports in any other study. It is very difficult to arrive at firm conclusions based on the present evidence, but as a suggestion for further work it may be mentioned that a systematic study could be made of momentum and heat transports and of different energy conversions based on observed winds and temperatures, on conventionally analysed maps, and on numerically analysed maps. Such an investigation would undoubtedly help to answer the questions concerning the possible systematic errors in the numerical analysis procedures and their effect on the energy conversion calculations.

4. SUMMARY

It has been attempted in this investigation to calculate the energy conversions between the four different forms of kinetic energy. Whenever possible we have divided the total energy conversion into a contribution from the quasi-non-divergent flow and a contribution from the divergent components of the flow.

One of the main results of the calculations is that the divergent components of the flow in general play a minor role in the different energy conversions. We have thus found a reconfirmation of the consistency of the quasi-geostrophic nature of the atmospheric flow.

A second major result is that the energy K_{SE} created by conversion from the eddy available potential energy A_E, is used to maintain the other three forms of kinetic energy K_{SZ}, K_{MZ}, and K_{ME} through direct conversions $C(K_{SE}, K_{SZ})$, $C(K_{SE}, K_{MZ})$, and $C(K_{SE}, K_{ME})$ of which the first conversion $C(K_{SE}, K_{SZ})$ is especially large. Only a small amount of energy finds its way back to K_{SE} through the catalytic conversion $C(K_{ME}, [K_{SZ}], K_{SE})$.

The third major result is that the conversion $C(K_{SZ}, K_{ME})$ is of almost the same magnitude as $C(K_{SE}, K_{SZ})$. It looks therefore as if the energy K_{SZ} converted for K_{SE} almost immediately is reconverted to K_{ME}, a process which explains why the barotropic waves (K_{ME}) have relatively large amounts of the total kinetic energy.

The barotropic zonal flow u_{MZ} has a relatively large amount of kinetic energy, K_{MZ}, but the energy exchange between K_{MZ} and the other energy reservoirs is comparatively small leading to a residence time of 2–3 weeks for this energy component.

The result obtained in the present observational study is in good agreement with the energy diagram obtained by Smagorinsky [7] from his basic general circulation experiment. Two exceptions are found: The energy conversion $C(K_{SZ}, K_{MZ})$ is positive in the numerical experiment, but negative in the observational study; and the energy conversion $C(K_{SE}, K_{ME})$ is small in the numerical experiment, but large in the observational study. The reason for the former discrepancy is most likely that the observational study only considers the region north of 20° N., and that the positive contribution from the Hadley cell is missing in the present study. The reason for the latter discrepancy is probably that the intensity of the eddies is too small in the numerical experiment including very small amplitudes on the planetary scale.

The calculations carried out in this study have been based on analysis of geopotential fields at selected isobaric surfaces. Our computations are therefore quasi-geostrophic, and we have made use of the divergence and vertical velocity implied by the quasi-geostrophic motion. It will be interesting to make similar calculations based on observed winds at a later time.

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